

# MathLinks

## **PROFICIENCY CHALLENGES GRADE 6 SETS 9-12**

Proficiency Challenges are sets of interesting, mixed-topic problems. It may take a lot of time to complete each set, so consider doing only one or two parts at a time.

## PROFICIENCY CHALLENGE 9

1. A cryptarithmic is type of puzzle where each letter stands for a different digit 0 through 9. In any given problem, the same letters represent the same digits. However, letters in different problems may have different values.

Try to solve the following cryptarithmetics. Problem a has multiple solutions. Problems b - d have only one solution each.

a.

$$\begin{array}{r} T O M \\ + N A G \\ \hline G O A T \end{array}$$

b.

$$\begin{array}{r} S E N D \\ + M O R E \\ \hline M O N E Y \end{array}$$

c. (Hint:  $L = 9$ .)

$$\begin{array}{r} W O W \\ W O W \\ + S O \\ \hline C O O L \end{array}$$

d.

$$\begin{array}{r} N E L S O N \\ + C A R S O N \\ \hline R E W A R D \end{array}$$

2. Which of the following pairs of expressions are equivalent? Explain your reasoning.

$4(3x - y) \text{ and } 12x - 4y$

$32 + 16y \text{ and } 8(4 + 2y)$

$3(x + 2y) \text{ and } 3x + 2y$

3. Consider these two expressions:  $2(3x - 9)$        $6x - 9$

Viviana thinks that the expressions are equivalent because  $2 \cdot 3x = 6x$ .

Critique Viviana's thinking and write an explanation or draw a diagram that could help her understand her mistake.

4. Create two expressions that are equivalent but do not look the same. At least one of the expressions must have parentheses. Both expressions must have more than three terms
5. Consider this inequality:  $n > 3$ . Choose all of the statements that can be represented by this inequality.
- |  |  |
|--|--|
| A. Natalia has no more than \$3.               | B. Walter lost three marbles.                  |
| C. Marcus has always had more than three pets. | D. The temperature increased by 3 degrees.     |
| E. Eldrick has less than three pets.           | F. Lettie has never had fewer than three pets. |

## PROFICIENCY CHALLENGE 10

1. Niki is sharing 4 slices of bread with her friends, Sam and Brianna. Niki's mom says she's only allowed to have one slice and must share the rest equally. How much bread do Sam and Brianna get each?
2. Vince and six friends are sharing 12 slices of bread. Each person takes one and one-half slices.
  - a. How much bread is left?
  - b. How much more bread would each person receive if they divided what's left equally between them?
3. You've been asked to help plan a field trip to the natural history museum. The teachers are trying to figure out how many buses they need to order. Here's some other information.
  - All 171 sixth graders in the school are going.
  - The museum requires at least one adult supervisor for every 12 students.
  - The bus company requires at least two adults on each bus.
  - Each bus can hold 30 passengers.

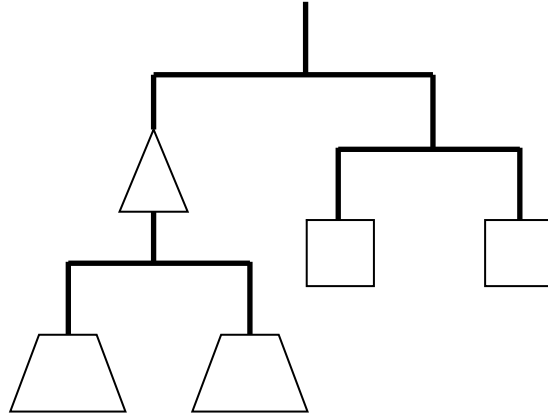
How many buses should be ordered to make this trip happen?

Make sure you explain how your answer meets all the conditions required by the museum and the bus company.

4. Find each unknown quantity.
  - a. A quantity plus itself is 15.  
What is the quantity?
  - b. One-fourth of a quantity is 12.  
What is the quantity?
  - c. A quantity plus  $\frac{1}{4}$  of itself is 15. What is the quantity?
  - d. A quantity minus  $\frac{1}{7}$  of itself is 12.  
What is the quantity?
5. Consider the following equation:  $4\left(\frac{2x-4}{6}\right) = 8$ . Try to find the solution using more than one method.

## PROFICIENCY CHALLENGE 10 (Continued)

6. The mobile below is balanced. Same shapes have the same weight. Different shapes have different weights. Use this information for the problems on this page. Each problem is independent of the others.



- What is the value of each shape if the total weight of the mobile is 36 and the triangle weighs 9 ounces?
- What is the value of each shape if the total weight of the mobile is  $4\frac{1}{2}$  ounces and the triangle weighs 1 ounce?
- What is the value of each shape if the total weight of the mobile is 1 ounce and the trapezoids each weigh  $\frac{1}{8}$  ounce?
- Juan and Marco were each trying to answer the following question: What is the value of each shape if the total weight is 24 ounces? They compared their solutions and found that they were different. Is it possible that they could both be right? Explain.
- Does the following statement appear to be true for different total weights of the mobile? Explain

$$\square - \text{trapezoid} > 3$$

## PROFICIENCY CHALLENGE 11

1. A geometric sequence is a sequence in which each term after the first term is obtained by multiplying (or dividing) the previous term by the same value.

State the multiplier (or “divider”) for each geometric sequence below.

- a. 6, 12, 24, 48 ...
- b. 18, 54, 162, 486 ...
- c. 100, 1000, 10000, 100000 ...
- d. 1024, 512, 256, 128 ...

2. Find the missing values in each of the geometric sequences below.

Then find the multiplier or divider. Explain your reasoning for each answer.

- a. 60, \_\_\_\_, 240, 480 ...
- b. 16, 8, \_\_\_\_, 2 ...
- c. \_\_\_\_, 12, 36, 108, \_\_\_\_ ...
- d. \_\_\_\_, \_\_\_\_, 16, 64, 256 ...

3. Explain why the following sequences of numbers are not geometric sequences.

- a. 3, 6, 9, 12 ...
- b. 24, 22, 20, 18 ...

**PROFICIENCY CHALLENGE 11 (Continued)**

4. Change **exactly two** of the numbers in this table to create a table where columns of numbers represent equivalent ratios.

	<b>Wendy</b>	<b>Xandra</b>	<b>Yonie</b>	<b>Zeke</b>
<b>bracelets completed</b>	10	50	20	15
<b>hours worked</b>	4	40	8	2

5. Use each of these numbers exactly once to create a table that contains rows of numbers that represent equivalent ratios.

6, 42, 24, 72, 11, 2, 14, 33

Do you think your table is the only solution? Explain your reasoning.

6. Create three different algebraic equations that have the following solution.

a.  $x = 3$

b.  $x = \frac{1}{2}$

7. Before Mr. Moneybags died, he wrote a “last will” to distribute his estate in this way:

- He would first leave half of his estate to his wife.
- Then he would leave \$100,000 to his daughter.
- Then he leaves half of what remains to his butler.
- The final \$30,000 is donated to a charity.

What was the value of his estate?

## PROFICIENCY CHALLENGE 12

1. Francine saves 10% of her monthly paycheck every month. However, the economy is slowing down and her company reduced her monthly paycheck from \$4000 to \$3200. Francine decides that she wants to keep saving the same amount of money each month as she did before the reduction. What percent of her new (reduced) monthly paycheck should she be saving now?
2. Mr. Allen's first period class has 30 students and they averaged 92% on the last math test. His second period class has only 15 students and they averaged 86% on the same test. What is the combined average for first and second period?
3. The average life expectancy for a person is about 75 years. How many seconds are in 75 years? (You can assume that leap years do not count.)

We spend about 30% of our time sleeping. How many seconds does the average person spend sleeping in their lifetime?

4. In the middle of Monday night, Brenda snuck downstairs and took 10% of the 100 cookies that were in the cookie jar. On Tuesday night, she took 20% of what was left. On the Wednesday night, she took 25% of what was left. On Thursday night, she took one-third of what was left.

How many cookies did she take on each night?

How many cookies were left in the jar on Friday morning?

5. In Sweeny's garage, there are several bicycles, tricycles, and wagons. In total there are 17 wheels. What are all the possible combinations of bicycles, tricycles, and wagons in Sweeny's garage? (You may assume that he has at least one of each type.)
6. What is 84% of 96% of 400?

Explain why the result above must be equal to the value obtained by finding 96% of 84% of 400.

7. Fill in the values for the blanks to make each statement true:

\_\_\_\_\_ is 30% of \_\_\_\_\_.

12 is \_\_\_\_\_% of \_\_\_\_\_.

\_\_\_\_\_ is \_\_\_\_\_% of 60.